



# Numerical Simulation of Convective Heat Transfer for In-Flight Icing

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## Abstract

Convective heat transfer is the primary mechanism during glaze ice accretion. Hence, modeling heat transfer over rough surfaces and the laminar-turbulent transition is essential in determining the final ice shape. The chapter presents the prediction of ice shapes via an improved treatment of the integral boundary layer. The onset of the laminar-turbulent regime is evaluated using an intermittency function to represent the transition region rather than an abrupt laminar-turbulent transition occurring at a point. An equivalent sandgrain roughness height and its consequent rough Stanton number correlation are evaluated. A modified thermal wall function) is presented that considers the effect of roughness height acting as a thermal insulator, countering the Reynolds number's effect of intensifying the heat transfer rates. The local heat transfer coefficient is calculated for a rough cylinder and icing airfoils. The results are compared with experimental data and open-source numerical simulations, showing a better prediction of ice shapes.

## Keywords

Ice accretion · Aircraft icing · Rough heat transfer · CFD · Freezing · Convective heat transfer · Integral boundary layer · Laminar-turbulent transition

## Introduction

In-flight ice accretion on aircraft components causes aerodynamic performance degradation, weight increase, and control and maneuver difficulties, leading to an operational safety margin reduction. When the aircraft is flying through a super-cooled water droplets cloud in a meta-stable thermodynamic equilibrium, ice accretion on some aerodynamic surfaces will occur if they are not adequately protected. Commercial aircraft have ice protection systems that guarantee safe flight in icing conditions. Numerical tools are used to analyze such ice protection systems' failure effects. In addition, it is a crucial tool to help engineers decide whether the airfoils must be protected.

Convective heat transfer is dominant in glaze ice formation with liquid water near freezing and is the primary mechanism for removing the solidification enthalpy. It has lesser influence as temperatures get farther below freezing, with rime ice forming instantaneously as the droplets impact the aircraft. (Gent et al. 2000) The heat transfer coefficient is the most critical and challenging parameter for accurate glaze ice shape prediction.

Usually, in classic icing codes, the heat transfer calculation of external flow is done by the integral boundary-layer analysis based on sandgrain roughness height  $k_s$ , and by considering abrupt laminar-turbulent transition. The  $k_s$  concept has some limitations because it comes from experimental data of flows in rough pipes and not external boundary layers.

There is experimental evidence (Havugimana et al. 2002; Kerho and Bragg 1997; Pimenta et al. 1975) that sandgrain-type integral analysis does not satisfactorily estimate the heat and momentum transfer of flows over rough surfaces.

Another reason to revisit airfoil icing heat transfer derives from previous works about airfoil thermal anti-ice simulation, which were carried out by (Silva 2002; Silva and Silves 2002; Silva et al. 2003, 2005).

The main conclusions of which are the following:

1. The boundary-layer integral analysis can provide satisfactory results if its assumptions are reviewed, and limitations are considered.
2. The laminar-turbulent transition region extent and intermittency shape significantly affect the heat and mass, and an abrupt transition may not best represent the actual phenomenon.

Calculating the heat transfer coefficient on airfoils with icing accretion is based on the integral boundary layer evaluation and is like the fundamental work of (Makkonnen 1985). It is considered a fully rough cylinder surface with an ice-like roughness of the pyramidal type. The heat transfer is modeled using Nikuradze's concept of equivalent sandgrain roughness height  $k_s$ .

Makkonnen (1985) proposed a calculation procedure for laminar and turbulent heat transfer between external flow over the rough surface of a cylinder. The author was interested in calculating the mean heat transfer coefficient for the circular cylinder to predict the mass of ice accreted on electrical power transmission lines. The author assumed the occurrence of a laminar-turbulent transition, where the flow passes from the laminar to the turbulent regime in an abrupt manner, i.e., without a transition region. The conclusion was that the proposed procedure was adequate to predict the mean Nusselt number and stated that the process does not precisely calculate the distribution of the local Nusselt number due to the following uncertainties:

1. The onset of laminar-turbulent transition may not be adequately predicted.
2. The model does not provide good results for the transitional flow region.

The model was developed with assumptions that limit the application to a wide range of operational conditions. Stefanini et al. (2007) implemented a model for the laminar-turbulent transition. It calculated the local convective heat transfer on an iced airfoil. The authors conclude that an abrupt transition model might not represent the phenomena under some icing conditions, mainly in the first-time steps of the simulation. Silva et al. (2003) developed a numerical simulation code to simulate anti-icing systems. They concluded that besides other effects, the laminar-turbulent transition onset and region extent significantly affects the heat and mass transfer

(Silva et al. 2007, b). The transition region modeling affects the airfoil's local heat and mass transfer prediction. It is relevant to determining the position at which water freezes in ice shape simulation or fully evaporates in an anti-icing simulation.

An evaluation of the boundary layer based on the work of Makkonen is implemented in this chapter, and the local heat transfer coefficient on a rough cylinder is estimated.

The present chapter uses an integral boundary layer evaluation for rough surfaces with a smooth laminar-turbulent transition model based on the intermittency concept, which is more realistic. The transition was represented as a region with a given length. The flow goes from a fully laminar to a fully turbulent regime and is linked by an intermittency function. The transition onset was established, and the convective heat transfer coefficient in the transition region was evaluated through a linear combination of the laminar coefficient and the turbulent coefficient weighted by an intermittency function  $\gamma(s)$ .

The local Nusselt number and heat transfer coefficient was calculated for rough cylinders, and results were compared with simulation results by Makkonen and experimental results by Achenbach (Makkonen 1985; Achenbach 1977).

During glaze ice accretion, convective heat transfer is the primary mechanism for removing solidification enthalpy, causing water to freeze. The mass of accreted ice largely depends on mean heat transfer, but the position where runback water freezes depend on the local heat transfer coefficient value. The convective heat transfer coefficient sharply increases when the laminar-turbulent transition occurs.

Other effects related to water film hydrodynamics, surface tension, beads formation, and rivulets flow produced changes in surface wetness and heat exchange between the water film surface and the environment. As the onset of water freezing relates to ice shape, it is essential to gauge the convective process as accurately as possible (Stefanini et al. 2008).

Due to icing tunnel operational limitations, convective heat transfer may differ significantly from that of in-flight operation, thus requiring an icing simulation tool to evaluate tunnel-flight similitude. Both turbulent skin friction and heat transfer increase. At the same time, the roughness affects the laminar-turbulent transition region onset position and its extension.

Rough surface effects on heat transfer and transition were studied in icing literature (Makkonen 1985; Poinsette 1989; Bragg et al. 1996; Fortin 2001), and their influence on glaze ice shape was recently studied (Stefanini et al. 2007; Stefanini 2009). His classic work (Makkonen 1985) used (Achenbach 1977) experimental data over a cylinder with an isothermal and rough surface to validate his boundary layer integral evaluation procedure.

Some of the classic two-dimensional airfoil icing prediction codes use a heat transfer calculation procedure based on an integral evaluation of the boundary layer, as shown in the works by LEWICE (Wright 1995), ONERA 2D (Guffond and Brunet 1988), TRAJICE (Gent 1990), and others (Makkonen 1985; Fortin 2001). These icing codes consider a smooth surface in a laminar regime and a fully rough surface in a turbulent regime. The laminar-turbulent transition occurs abruptly downstream of its onset position. Those works have similarities to the work of

(Makkonnen 1985). Their models use an integral analysis, and ice roughness height is used to predict the transition onset position and the momentum and heat transfer enhancement effects.

All the authors mentioned above also use Nikuradze's concept of equivalent sandgrain roughness height  $K_s$ . This approach was criticized by Pimenta et al. (1975) because one sole parameter represents three independent roughness characteristics like height, distribution, and shape. Recently, Stefanini et al. (2010) showed that two new parameters are needed to represent Achenbach's close-packed pyramidal roughness (Achenbach 1977) through Owen's  $St_k$  correlation (Owen and Thomson 1963).

There is a clear advantage to using a laboratory-controlled experiment with a precise definition of  $K_s$  (Achenbach 1977) in validating the heat transfer prediction of integral or differential icing simulation codes.

Differential analysis application to aircraft icing problems momentum is not as common as integral analysis. Results presented by the California State University Long Beach group have been validated concerning predicting momentum and heat transfer around icing airfoils (Cebeci 1989; Shin et al. 1992; Cebeci et al. 1992). These works also used Nikuradze's concept of equivalent sandgrain roughness height  $K_s$  in eddy viscosity formulation. Despite being differential RANS, the codes are parabolic boundary-layer type codes.

The present authors have not found works on Computational Fluid Dynamics (CFD) or on elliptical codes applied to icing simulation that are specifically validated against convective heat transfer experimental data, except for the comprehensive work of (Goldberg and Batten 2017).

Regarding anti-ice thermal performance, the location of water evaporation may change. Peak temperatures vary locally, whereas average temperatures vary globally (Silva et al. 2006, 2007, 2008b, 2009; 2015; Silva 2009; Stefanini et al. 2010; Al-Khalil et al. 2001). In terms of ice accretion, variations may cause water to freeze by removing the enthalpy for the water solidification (Stefanini et al. 2008). This changes the ice's shape and may change its local features, such as the position and height of its horn.

In some cases, the 3D wing or ice shapes may be approximated into their 2D profiles by similitude, as assumed by classic anti-icing codes (Makkonnen 1985; Wright 1995; Guffond and Brunet 1988; Gent 1990). This allows the analyst to use 1D integral solutions for the boundary layer and simplify the skin friction and heat transfer coefficient calculation. This approach is not always possible due to its inherent limitations. Still, when applicable, it expedites the design process by using analytical and integral solutions instead of more demanding computational methods.

Even though numerical integral solutions are not as novel as the numerical differential Reynolds-Averaged Navier-Stokes (RANS) solutions, they still play a critical, crucial role in equipment design and aircraft certification. Ordinary RANS solvers are the 2D finite-difference boundary layer codes and 3D finite-volume CFD tools. The latter is significant when simulating complete configurations of an aircraft wing, pylon, and fuselage as well as its swept wings, multi-body wings, air data

probes, antennas, air inlets, and other devices installed on the aircraft's external surfaces.

There are still advantages to using integral solutions in the field of icing:

1. Integral solutions or semi-empirical formulas are used in validated icing tools such as LEWICE (Wright 1995), ONERA2D (Guffond and Brunet 1988), and TRAJICE (Gent 1990).
2. Authorities accept integral solutions in the process of aircraft certification for icing.
3. Integral solutions can be used as verification when simulating CFD cases for which no experimental data is available.
4. Engineers can produce hundreds of integral solutions quickly and with sufficient accuracy.

The latter is crucial for trade-off studies of a complete critical case test matrix considering several flight or tunnel conditions. On the other hand, integral solutions have limitations compared to RANS CFD because they are one-dimensional, assume local properties, do not predict heat transfer in detached boundary layer regions, and neglect most flow history effects. Therefore, they have limited application to 3D complex shapes or multi-body configurations, and most consider an incompressible flow.

Momentum boundary layer modeling may affect thermal boundary modeling because the momentum thickness  $\delta_2$  and freestream turbulence level  $Tu$  define the onset location and length of the laminar-turbulent transition. This approach is useful helpful for application in entirely or partially clean airfoils.

For icing airfoils, the sand-grain roughness height  $k_s$  is a vital parameter to define the transition onset location (Stefanini et al. 2010):

$$\text{Re}_{K_s} = \frac{u_e \cdot K_s}{\nu} > 600 \quad (1)$$

The boundary layer edge velocity  $u_e$ . The pressure coefficient around the airfoil or body is estimated by considering potential flow. Equation (1) is not universally applicable for all icing problems; it does not apply, for instance, when roughness height is more significant than boundary layer thickness.

Some icing models, which do not solve the energy boundary layer equation in a turbulent regime, use momentum for heat transfer analogy to correlate turbulent skin friction  $C_f$  with turbulent Stanton  $St$ . This analogy is standard when solving flow over isothermal and rough surfaces such as those of ice shapes (Makkonnen 1985; Stefanini et al. 2010), for instance:

$$St_{turb,rough} = \frac{C_{f,rough/2}}{Pr + \frac{\sqrt{C_{f,rough/2}}}{St_k}} \quad (2)$$

$St_k$  represents the roughness sub-layer thermal resistance in series with the main rough, turbulent boundary layer's thermal resistance. In addition,  $C_{f,rough}$  is a

function of sandgrain height  $k_s$  and momentum thickness  $\delta_2$  over smooth surfaces (Guffond and Brunet 1988).

On the other hand, some anti-icing models, such as those implemented by previous authors (Silva et al. 2006; Al-Khalil et al. 2001), require estimation of  $C_f$  distribution to obtain adequate water runback hydrodynamics. The shear near the wall  $\tau_0$  defines the runback movement and height and the breakdown process from water film to water rivulets.

Several analogies are available for different applications, but the simplest is the Reynolds analogy, which directly correlates  $C_f$  with  $St$ :

$$St = \frac{C_f}{2} \quad (3)$$

In addition, the pressure gradient and its variation may affect transition prediction and the  $C_f$  distribution (Silva et al. 2015). Pressure variation or flow acceleration effects are not negligible when simulating flow around cylinders or actual thick and non-symmetric airfoils or over some rime or glaze ice shapes.

Recently, some authors developed their models (Langtry et al. 2006) or updated commercial codes (Chakravarthy et al. 1998; Goldberg and Batten 2017), while others validated CFD turbulence models that use intermittency transport models with experimental data (Tobaldini Neto et al. 2008; Halila et al. 2015).

Because laminar-turbulent transition and turbulence models should not be generalized, using a CFD tool to simulate  $C_f$  or, ultimately,  $St$  around thermally protected airfoils still requires validation with experimental data. The main reason for this is that models are relatively new, and there is a lack of experimental measurement of local  $C_f$  and  $St$  for different geometries and conditions, especially for rough icing surfaces.

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## Previous Works

Field (Pimenta et al. 1975) states that turbulent heat transfer over rough surfaces may depend on roughness height, shape, and distribution. However, most classic works tried to identify the surfaces and describe their performance with a single general parameter. Friction results from turbulent pipe flows have usually been extended to boundary-layer flows over plates. This was the approach of Schlichting and Prandtl in 1934 (Pimenta et al. 1975) when they conceived the sandgrain roughness  $k_s$  concept. The  $k_s$  value may be a fraction of the actual roughness height because it must also represent roughness shape and distribution effects. Some authors performed rough pipe flow experiments and proposed two-layer models to predict heat transfer:

1. The first layer is very thin, close to the wall, and concentrates all the effects caused by the distributed protuberances.
2. The second is located above the first and behaves as a “fully turbulent layer.”

Schlichting and Prandtl assumed the Reynolds analogy was valid, using turbulent Prandtl number  $Pr_{turb}$  value, or eddy-diffusivity distribution to match both layers. This procedure was applied by Dipperey and Sabersky (1963) and Owen and Thomson (1963), and this approach is used in icing literature.

Makkonnen (1985) proposed a calculation procedure for laminar, transitional, and turbulent heat transfer between an external flow and the rough surface of an iced cylinder. The author implemented a laminar boundary-layer conduction thickness for evaluation with (Smith and Spalding 1958) model. The turbulent Stanton number  $St_{turb}$  requires a friction coefficient  $C_f$  obtained from momentum thickness  $\delta_{2,turb}$ , plus turbulent Prandtl number  $Pr_{turb}$  and experimental parameters. Therefore, the turbulent heat transfer coefficient –  $h_{turb}$  – is based on a fully rough law of the wall, a two-layer model with empirical adjustments, and on sand-grain definition (Dipperey and Sabersky 1963; Kays and Crawford 1993; Owen and Thomson 1963). Both laminar  $h_{lam}$  and turbulent  $h_{turb}$  coefficients are evaluated through the analogy between momentum and heat transfer, which assumes a flow over a near isothermal surface with no mass transfer. Makkonnen (1985) considered that an abrupt laminar-turbulent transition occurred, and that the momentum thickness had no discontinuity at the transition point.

Based on experimental observations, (Pimenta et al. 1975) proposed a law of the wall and a mixing-length turbulence model to be used in finite difference boundary-layer codes. The author found that mixing-length theory results were closer to experimental data than classical correlations or integral methods when estimating the effect of roughness and transpiration on turbulent flow and heat transfer.

Cebeci (Cebeci 1987, 1989; Havugimana et al. 2002) applied his finite-difference boundary-layer code to improve the heat and mass transfer prediction around airfoils contaminated by liquid water or ice. The Cebeci-Smith mixing-length turbulence model was adjusted to represent the flow over roughness, and this turbulence model was validated by Shin et al. (1992).

(Havugimana et al. 2002; Thwaites 1949) compared skin friction and heat transfer over rough plates to experimental data in the literature. The authors used a modified Cebeci-Smith mixing-length turbulence model that considered sandgrain and discrete element roughness models. They concluded that classical boundary-layer integral analysis predicts heat transfer compared to experimental data results.

The classic icing codes LEWICE (Macarthur et al. 1982), TRAJICE2 (Cansdale and Gent 1983), and ONERA2D (Guffond and Brunet 1988) estimate ice shapes over non-protected airfoil surfaces. A comprehensive review of the mathematical models and comparing these codes' prediction capabilities were published (Wright 1995).

At British Royal Aircraft (Cansdale and Gent 1983) implemented a pioneering work regarding thermal balance around non-heated airfoils under icing conditions by extending (Messinger 1953) mathematical model to compressible flow and water vapor local concentration. Gent (1990) implemented the numerical code TRAJICE2, which predicts two-dimensional ice shapes on airfoils. The author approximated the flow over the airfoil leading edge as if it were the flow over the front of a cylinder and, by scaling experimental results of heat transfer around rough cylinders,



**Table 1** Transition parameters of Stefanini et al.

Parameter	Upper surface	Lower surface
Onset position $s_O/c$	0.003	-0.001
End position $s_E/c$	0.041	-0.037
Sandgrain height $k_s$ (m)	0.00023	

developed an empirical expression to evaluate the convection heat transfer coefficient on the airfoil surface. Alternatively, (Gent 1990) implemented a boundary-layer integral analysis like (Makkonnen 1985) to assess the laminar and turbulent heat transfer coefficient over a near isothermal surface with no mass transfer effects. The laminar to turbulent transition is assumed to occur abruptly when the Reynolds number on sandgrain roughness is  $Re_k > 600$ . As observed with other classic icing codes, heat transfer prediction is only valid for thin ice accretions, i.e., at the beginning of the accretion process, without a flow separation (Gent et al. 2000) (Table 1).

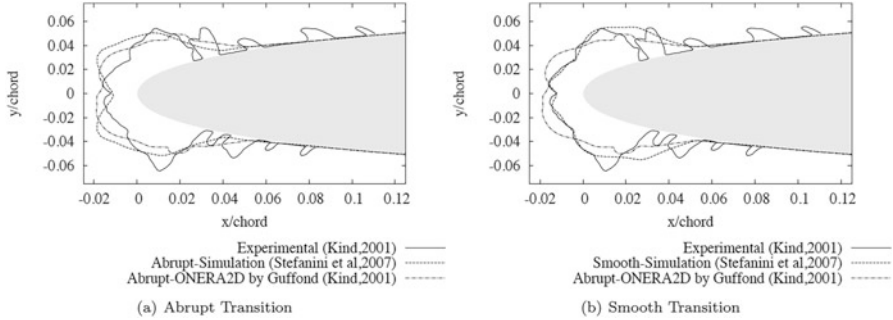
The LEWICE code developed by researchers (Macarthur et al. 1982); (Ruff and Berkowitz 1990); (Wright 1995) from NASA estimates the potential flow around the airfoil by a panel method, considering collection efficiency, momentum and thermal boundary-layers as well as ice shape. For convection heat transfer calculation, this code estimates laminar boundary-layer conduction thickness  $\Delta_{4,lam}$ , assumes that transition criteria triggered by roughness  $Re_k > 600$ , where  $v_k$  is the velocity at the top of the roughness element and estimates the turbulent heat transfer coefficient  $h_{turb}$  over a rough surface with similar assumptions and procedures as those adopted by Makkonnen (1985). Ice growth is predicted by LEWICE's thermal module that adopts (Messinger 1953) equations for the freezing process over an adiabatic airfoil surface.

Guffond and Brunet (1988) developed the ONERA2D code at the Office National D'Études et de Recherches Aéropatiales – ONERA, France, to estimate ice geometry. Unlike LEWICE, it solves the full potential flow around clean or iced airfoils in a C-type mesh using the finite element method. With the pressure field, ONERA2D calculates water droplets' trajectories, collection efficiency, convective heat transfer coefficient, and thermal balance (Messinger 1953) to estimate ice shape.

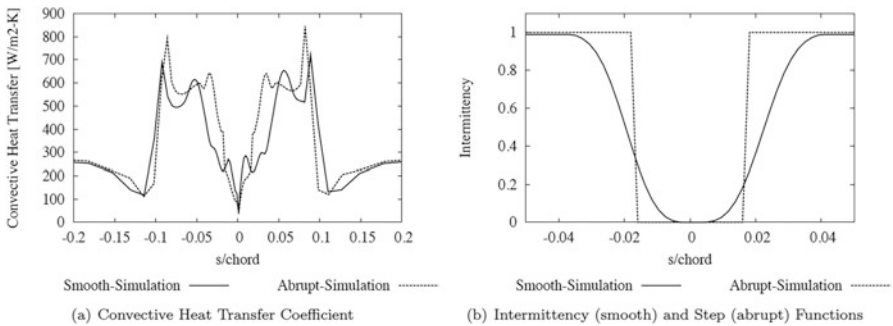
The ice growth modules of the classic icing codes have similar mathematical models and numerical implementation structures. Despite limitations in predicting the glaze ice growth process, their numerical precision is well known, and the authorities accept their ice-shape results.

Several working groups compared numerical results with experimental ice shapes. The Applied Vehicle Technology Panel, Research and Technology Organization, North Atlantic Treaty Organization – AVT-NATO-RTO (Kind 2001) performed the most recent benchmarking work, which made experimental data from various ice tunnels and numerical results available, involving codes widely used by the industry and by academia, which uses abrupt transition.

Stefanini et al. (2007) changed boundary layer parameters in ice-shape simulations. The reference case was simulated using these parameters and an intermittency function. Ice shape results were like the experimental ice shape (Kind 2001). Figures 1 and 2 show ice shape results, and Table 1 shows boundary layer parameters.



**Fig. 1** Comparison of Stefanini et al. results with the abrupt transition model



**Fig. 2** Heat Transfer coefficient and Intermittency – comparison between Stefanini et al. and classical icing code

Li and Paoli (2019) developed a 3D icing shape simulation on an ONERA M6 wing. It used (Stefanini et al. 2010) rough heat transfer correlations and the thermal wall function developed by Silva et al. (2011) for OpenFoam. This is part of a project to create a hybrid RANS-LES solver to simulate icing.

Yassin et al. (2021) developed a CFD simulation over wind turbine blades contaminated with icing. The ice shapes were obtained experimentally, and the CFD was run to determine the performance degradation by verifying  $C_p$  and  $CL$  results. The authors used the boundary layer momentum model suggested by Silva et al. (2011), which provided the best results compared to experimental data.

## Icing Numerical Code

Field (Stefanini et al. 2007) described the ONERA2D code, implemented by Guffond and Brunet (1988), chosen as the numerical icing tool for predicting ice shapes. ONERA2D uses a predictor-corrector scheme that estimates the ice growth process in two runs: the first run considers a clean airfoil geometry and total duration, and the second uses the first ice shape as the new airfoil surface to simultaneously