

Boundary-Layers Integral Analysis - Airfoil Icing

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Various classic icing codes have been developed and used by the aircraft manufacturers to predict the ice shapes evolution in non-protected airfoil surfaces. These codes apply boundary layer integral analysis, based on sand grain roughness, to estimate the convective heat transfer coefficient around the iced airfoil and adopt abrupt transition between laminar and turbulent flow. In the present work, the integral analysis is also used but a smooth laminar-turbulent transition model, based in the intermittency concept, is included. The transition onset position is estimated by empirical correlations or just imposed. The ice accretion on a NACA 0012 airfoil is simulated with a new convective heat transfer routine, which assumes smooth laminar-turbulent transition, and the results are compared with the ice shapes generated by ONERA2D original code and with experimental data.

Nomenclature

C	specif heat at constant pressure, $J/(kg \cdot K)$
c	airfoil chord, m
h	heat-transfer coefficient, $W/(m^2 \cdot K)$
k_s	sand grain roughness height, m
LWC	liquid water content, kg/m^3
Re_k	Reynolds number based on sand grain roughness height, $u \cdot k_s/\nu$
Re_s	Reynolds number based on the chordwise length, $u \cdot s/\nu$
Re_{δ_2}	Reynolds number based on momentum boundary layer thickness, $u \cdot \delta_2/\nu$
s	chordwise length, m
St	Stanton number, $h/(u \cdot \rho \cdot C)$
T	temperature, K
u	velocity, m/s

Subscripts

∞	free stream flow conditions
E	transition end position

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e	boundary layer
lam	laminar
O	transition onset position
$turb$	turbulent

Symbols

δ_2	momentum boundary layer thickness, m
ν	kinematic viscosity, m^2/s
ρ	fluid density, kg/m^3

I. Introduction

THE convective heat transfer is important at glaze ice formation conditions, when liquid water is near freezing, because the heat convection is the main mechanism to remove the solidification enthalpy. In the aerospace icing community, the most used heat transfer calculation procedure is the integral boundary-layer analysis based on sand grain roughness height k_s and abrupt laminar-turbulent transition. The reason to revisit airfoil icing heat transfer came from previous works about airfoil thermal anti-ice simulation, which were carried-out firstly by Silva,¹ summarized by Silva and Silves,² published by Silva et al.^{3,4} and recently extended by Silva et al.⁵⁻⁷ The main conclusions were: 1)The boundary-layer integral analysis can provide satisfactory results, if its assumptions are reviewed and limitations are considered; 2)The laminar-turbulent transition affects the heat and mass transfer significantly and an abrupt transition may not represent the real phenomena.

This paper present a integral boundary analysis applied to icing airfoils. The approach proposed is similar to the described by Makkonen.⁸ However, it has a new laminar-turbulent transition model, which is smooth rather than abrupt, and an alternative method to predict the sand grain roughness height.

II. Previous Works

According to Pimenta,⁹ the turbulent heat transfer over rough surfaces may depend on roughness size, shape and distribution. The friction results of rough pipe flows have usually been extended to boundary-layer flows over plates. Basically, this was the approach of *Schlichting* and *Prandtl* in 1934⁹ when they conceived the sand grain roughness k_s concept. The k_s value can be a fraction of the actual roughness height because it must also represent the effects of roughness shape and distribution.

Makkonen⁸ proposed a calculation procedure for laminar, transitional and turbulent heat transfer between external flow and the rough surface of an iced cylinder. Both laminar h_{lam} and turbulent h_{turb} coefficients are evaluated by the analogy between momentum and heat transfer, which assumes flow over a near isothermal surface without mass transfer. Makkonen⁸ assumed the occurrence of an abrupt laminar-turbulent transition and that the momentum thickness has no discontinuity at transition point.

Havugimana et al.¹⁰ compared the skin friction and heat transfer over rough plates to some literature experimental data. They concluded that classical boundary-layer integral analysis predicts the heat transfer when compared to experimental data and their results.

The classic icing codes LEWICE,¹¹ TRAJICE2¹² and ONERA2D¹³ estimate ice shapes over non-protected airfoil surfaces. A comprehensive review of the mathematical models and a comparison of these codes prediction capabilities were published by Wright et al.¹⁴

Gent¹⁵ implemented in the TRAJICE2 a boundary-layer integral analysis, which is similar to Makkonen,⁸ to evaluate the laminar and turbulent heat transfer coefficient over a near isothermal surface and without mass transfer effects. The laminar to turbulent transition is assumed to occur abruptly when *Reynolds* number based on sand grain roughness is $Re_k = u_e \cdot k_s / \nu > 600$, where u_e is the velocity at the boundary layer top edge.

LEWICE code, for convection heat transfer calculation, assumes a transition criteria triggered by roughness $Re_k = u_k \cdot k_s / \nu > 600$, where u_k is the velocity at the top of the roughness element, and estimates turbulent heat transfer coefficient $h_{\infty,turb}$ over a rough surface with similar assumptions and procedures adopted by Makkonen.⁸

ONERA2D code also uses a boundary-layer integral model similar to Makkonen.⁸ The laminar to turbulent transition is the same assumed by TRAJICE2.

The ice growth modules of the classic icing codes have similar mathematical models and numerical implementation structure. Despite the limitations to predict glaze ice growth process, the numerical precision and the ice shape results are accepted by the aerospace community.

There are several working groups that compared numerical results to experimental ice shapes. The most recent benchmarking work was performed by the Applied Vehicle Technology Panel, Research and Technology Organization, North Atlantic Treaty Organization - AVT-NATO-RTO,¹⁶ which made available experimental data from various ice tunnels and numerical results from mostly used codes by industry and academia.

Stefanini et al.¹⁷ changed the boundary layer parameters (s_O, s_E, k_s) for ice shape simulations. The same reference case of the present paper was simulated with these parameters and a ice shape result was similar with the experimental ice shape.¹⁶ Figures 1 and 2 show the ice shape results and table 1 shows the boundary layer parameters used in the simulations.

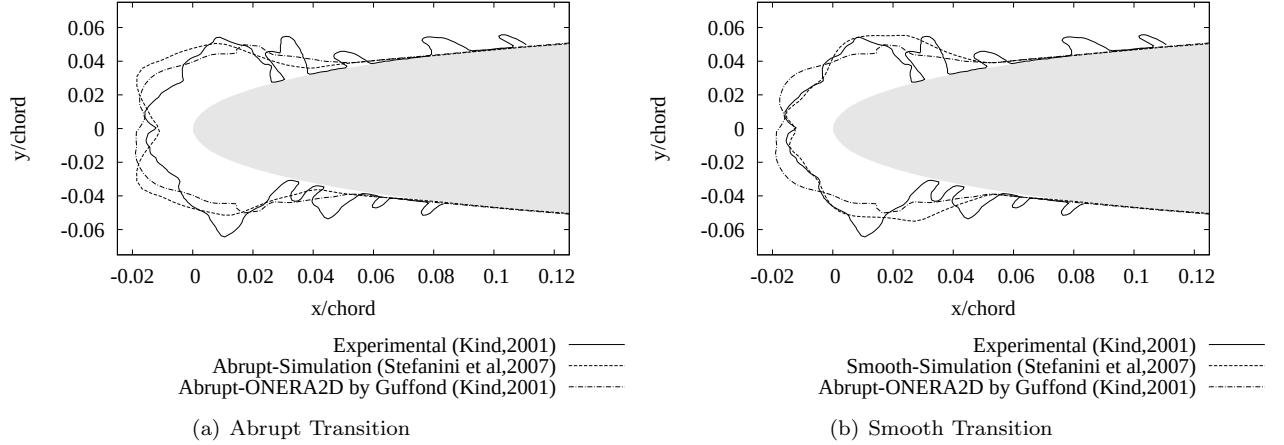


Figure 1. Simulation Results Compared to Baseline ONERA2D¹⁷

Table 1. Simulation Parameters¹⁷

Parameter	Upper Surface	Lower Surface
onset position s_O/c	0.003	-0.001
end position s_E/c	0.041	-0.037
sand grain height k_s (m)	0.00023	

III. Objectives

The objective of the present paper is: 1) assess the sensitivity of the ice geometry to the modifications on the heat transfer model proposed by Stefanini et al.¹⁷

IV. Icing Numerical Code

The ONERA2D code, implemented by Guffond and Brunet,¹³ was chosen as the icing numerical tool to predict ice shapes in the present paper. However, the original airfoil iced surface heat transfer evaluation module was replaced by a new one based on Makkonnen⁸ model.

Two laminar-turbulent transition models are used: 1) the abrupt one, which has the onset position predicted by empirical correlation and alters from laminar to turbulent regime at the transition point; 2) the smooth one, which has an intermittency function that smoothly links the laminar and turbulent flows throughout a finite transition region.

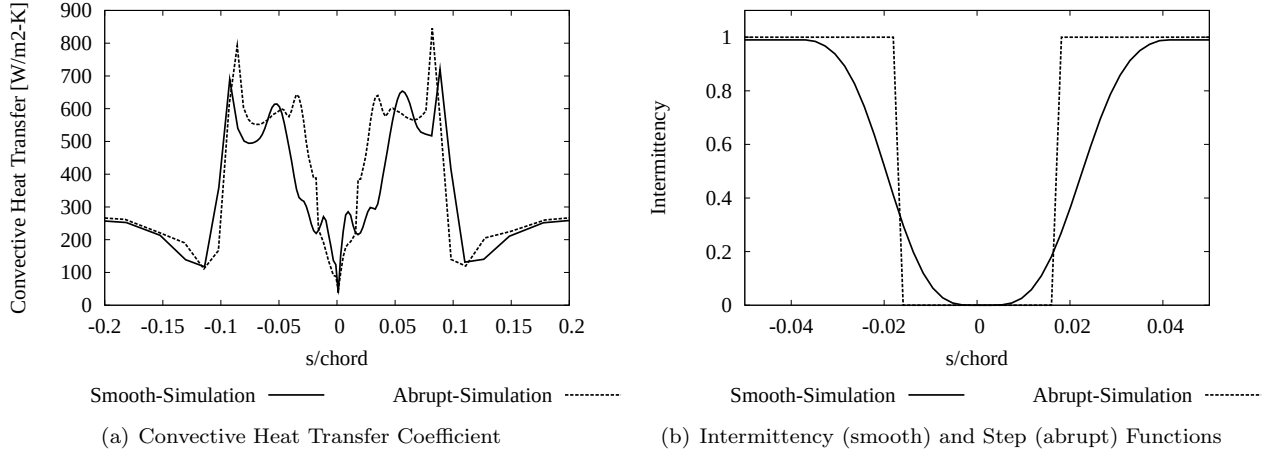


Figure 2. Simulation Results for Smooth and Abrupt Transition¹⁷

V. Characteristics of Proposed Boundary Layer Model

A. Sand Grain Roughness Height

A empirical correlation for the sand grain roughness height was used by LEWICE code and was presented by Ruff et al.¹⁸ The correlation uses experimental data to correlate the effects of the liquid water content, static temperature and velocity on the shape of the ice accretion formed. The resulting data are normalized by the airfoil chord c of 0.3 m. The k_s/c_0 used in the equations was 0.00117m.

Freestream Velocity (u_∞):

$$\frac{k_s/c}{k_s/c_0} = 0.4286 + 0.0044139 \cdot u_\infty \quad (1)$$

Liquid Water Content (LWC):

$$\frac{k_s/c}{k_s/c_0} = 0.5714 + 0.2457 \cdot LWC + 1.2571 \cdot LWC^2 \quad (2)$$

Static Temperature (T_s):

$$\frac{k_s/c}{k_s/c_0} = 46.8384 \cdot \frac{T_s}{1000} - 11.2037 \quad (3)$$

The value of sand grain roughness height is calculated by equation (4).

$$k_s = \left[\frac{k_s/c}{k_s/c_0} \right]_{u_\infty} \cdot \left[\frac{k_s/c}{k_s/c_0} \right]_{LWC} \cdot \left[\frac{k_s/c}{k_s/c_0} \right]_{T_s} \cdot k_s/c_0 \cdot c \quad (4)$$

B. Laminar-Turbulent Transition

It is difficult to predict the laminar-turbulent transition onset position and extension over irregular and rough surfaces because there is no general theory to describe the transition mechanisms. Therefore, all classic icing codes estimate transition onset position by empirical arguments.

ONERA2D¹³ adopts a empirical classical criteria to determine the onset transition position s_O that is given by:

$$Re_k = \frac{u_e \cdot k_s}{\nu} > 600 \quad (5)$$

Abu-Ghannam and Shaw¹⁹ developed empirical correlations to predict the beginning and the end of the transition region. The correlations are based on experimental data for a smooth flat plate under various velocity regimes and turbulence intensities tu . The end of the transition region given by the Equation 6.

$$Re_{\delta_2,E} = 540 + 183.5 \cdot (Re_L \cdot 10^{-5} - 1.5) (1 - 1.4\lambda) \quad (6)$$

where:

$$Re_L = 16.8 \cdot (Re_{s,O})^{0.8} \quad (7)$$

The intermittency function is zero $\gamma = 0$ in the region upstream the initial position (onset) s_O of transition region; and it is almost unity $\gamma = 0.99$ at the end position of transition region, where the flow regime becomes fully turbulent. Based in a comprehensive set of experimental data, Abu-Ghannam and Shaw¹⁹ defined the flow intermittency $\gamma(s)$ as:

$$\gamma(s) = 1 - \exp \left[-5 \cdot \left(\frac{s - s_O}{s_E - s_O} \right) \right] \quad (8)$$

where s_O is the transition onset position from stagnation point, s_E is the end position of transition region. Thus, the difference $s_E - s_O$ gives the length of transition region. The Stanton number in transition region is evaluated by linear combination of the St_{lam} and St_{turb} weighted by intermittency function:

$$St(s) = [1 - \gamma(s)] \cdot St_{lam} + \gamma(s) \cdot St_{turb} \quad (9)$$

VI. Reference Case

Kind¹⁶ published results of a comprehensive comparison between icing codes and experimental data measured at several test conditions and icing tunnels around the world. In this work, the case C13 , performed with a NACA 0012 airfoil profile in the Icing Research Tunnel at NASA Glenn Research Center, was selected as the reference case to simulate the ice growth. The conditions of the tests are presented in the table 2.

Table 2. Experimental Conditions

Flow Condition	Value	Flow Condition	Value
Angle of Attack (deg)	0.0	Liquid Water Content (g/m^3)	0.65
Freestream Velocity (m/s)	67	Median Volumetric Droplet Diameter (μm)	40
Freestream Static Pressure (Pa)	97147	Freestream Static Temperature (K)	264.4
NACA 0012 chord (m)	0.533	Duration (s)	672

VII. Results

Sand grain roughness height value is a parameter that influences significantly the ice shape result. The correlation for the prediction of the roughness height k_s , shown in the Equation (4), was implemented in the boundary layer sub-program proposed in the present paper. The reference case was simulated with the calculated k_s and with the boundary layer parameters presented in the Table 1. The results are shown in the Figure 3.

The value calculated by the Equation (4) was $2.19 \cdot 10^{-4}m$, which was very close to the value of k_s presented in the Table 1. The ice shape and heat transfer coefficient results were, consequently, very similar with the results of Stefanini et al.¹⁷

Equation (4) was widely used in icing codes and provided acceptable values of sand grain roughness height. The similarity between the results show that the values of k_s imposed by Stefanini et al.¹⁷ are coherent with the roughness height of the experimental ice shapes.

From the simulations presented in the Figures 1 and 2 it can be concluded that the use of a intermittency function make the convective heat transfer in the transition region more realistic. The intermittency function joins smoothly the onset position and the end transition position. In the present paper the onset position was given by the Equation (5). The end position of the transition region was given the Equation (6).

Figure 4 shows the ice shape result of the present paper simulation with the intermittency function. Table 3 show the boundary layer parameters used in this simulation. Similarly to the results of Stefanini et al,¹⁷

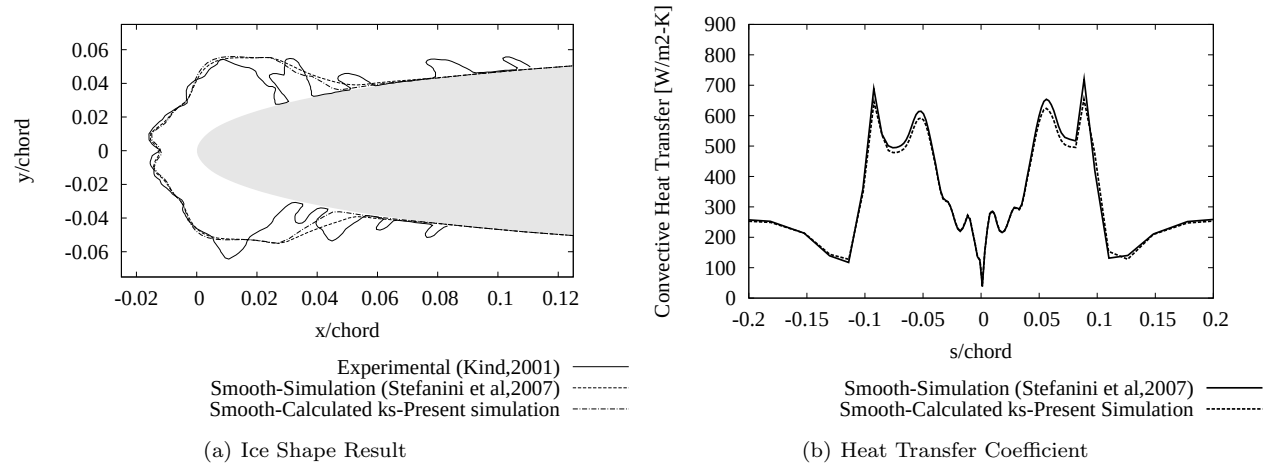


Figure 3. Simulations results for calculated k_s

the introduction of the smooth laminar-turbulent transition with a intermittency function provided more realistic ice shape for the present case simulation. The smooth increase of the heat transfer coefficient in the transition region contributes to change the position of the ice horn. In the smooth simulation it is dislocated downstream the chordwise and it has a rounded shape. The ice shape tends to fit the front experimental shape, but has deviations in the height of the ice shape.

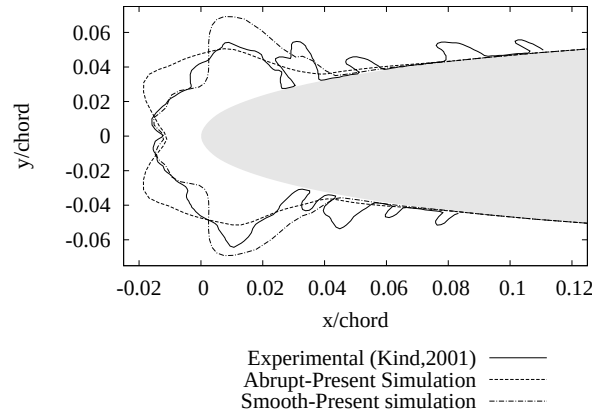


Figure 4. Present Paper ice shape results

Table 3. Present Paper Simulation Parameters

Parameter	Upper Surface	Lower Surface
onset position s_o/c	0.0117	-0.0116
end position s_e/c	0.0329	-0.0327
sand grain height k_s (m)	0.00022	

Figure 5 shows the heat transfer coefficient and the intermittency functions for the simulations of the present paper and of the Stefanini et al.¹⁷

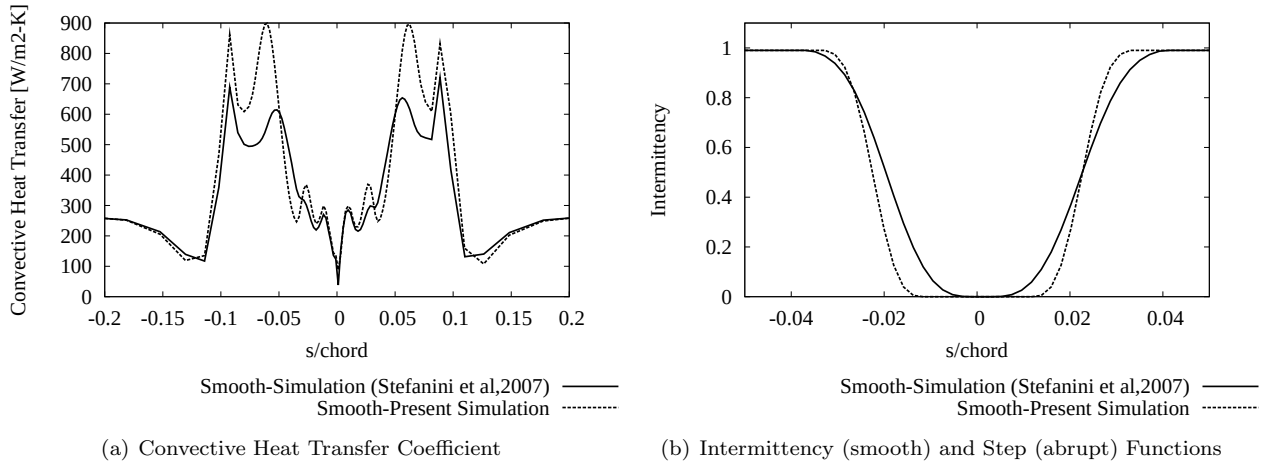


Figure 5. Comparison between the present and Stefanini et al.¹⁷ Simulations

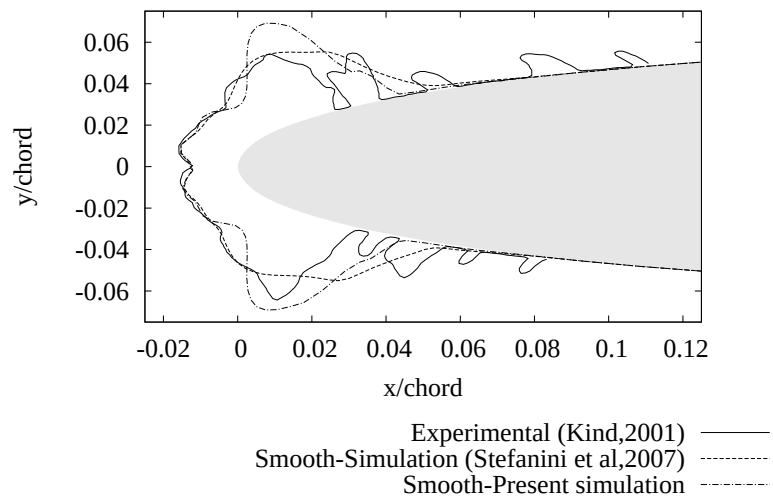


Figure 6. Comparison between the present and the Stefanini et al.¹⁷ simulation

VIII. Conclusion

An alternative k_s models was implemented in the heat transfer model developed by Stefanini et al.¹⁷ and the results were compared with the work of the same authors. However, the model based on the experimental data provided values of k_s very close to the fitted by Stefanini et al.¹⁷

The prediction of the transition length by Abu-Ghannam and Shaw¹⁹ correlations were used together with the classical onset prediction models and show that the ice shape result can be improved with the use a smooth than abrupt laminar-transition function. With this evidence, other models of transition length can be implemented and compared with the results shown in the present work.

From the results presented, it can be concluded that the use of a intermittency function make the convective heat transfer in the transition region more realistic. The use of an abrupt transition produced ice shapes with horn shapes near the onset position, which is not seen in experimental ice shapes.

In spite of the simple transition model used and the limitations imposed by the assumptions of integral approach, the presented results are encouraging and demonstrate that the study of methods of predictions and development of boundary layers are required for the subject.

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